B.Sc. Part II (Hons.) 4th Paper

(LPD)

Q: Solve $\frac{d^4y}{dx^4} - m^4 y = \sin mx.$

Soln. The given equation

$$(D^4 - m^4)y = \sin mx$$

For CF, $D^4 - m^4 = 0 \Rightarrow$

$$\Rightarrow (D^2+m^2)(D^2-m^2) = 0$$

$$\Rightarrow D^2+m^2 = 0 \Rightarrow D = \pm mi$$

$$\text{or } D^2 - m^2 = 0 \Rightarrow D = \pm m.$$

$$\therefore CF = Ae^{mx} + Be^{-mx} + C e^{mix} + De^{-mix}$$

$$\Rightarrow CF = Ae^{mx} + Be^{-mx} + E\cos mx + F\sin mx$$

For PI

$$PI = \frac{1}{D^4 - m^4}\sin mx$$

$$\Rightarrow PI = \frac{1}{(D^2+m^2)(D^2-m^2)}\sin mx$$

$$\Rightarrow PI = \frac{1}{D^2+m^2} \cdot \frac{1}{D^2-m^2}\sin mx$$

$$\Rightarrow PI = \frac{1}{D^2+m^2} \sin mx \cdot \frac{1}{-m^2-m^2}$$

$$\Rightarrow PI = -\frac{1}{2m^2} \cdot \frac{1}{D^2+m^2} \sin mx$$

$$\Rightarrow PI = -\frac{1}{2m^2} \times \frac{x}{2} \int \sin mx \, dx$$

$$\Rightarrow PI = -\frac{1}{4m^2} \cdot x \frac{(-)\cos mx}{m}$$

$$\Rightarrow PI = \frac{x}{4m^3} \cos mx.$$

Hence, the complete solution is given by

$$y = CF + PI$$

$$\Rightarrow y = Ae^{mx} + Be^{-mx} + E\cos mx + F\sin mx + \frac{x}{4m^3}\cos mx.$$